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ON THE HOCHSCHILD COHOMOLOGY OF ALGEBRAS WITH SMALL HOMOLOGICAL DIMENSIONS

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RÉSUMÉ. Nous étudions la cohomologie de Hochschild d'une algèbre A satisfaisant la propriété suivante : il existe un entier positif n_0 tel que la longueur de chaque chemin de ind A d'un module injectif vers un module projectif est bornée par n_0.

ABSTRACT. We study the Hochschild cohomology of an algebra A which satisfies the following property: there exists a positive integer n_0 such that the length of any path in ind A from an injective to a projective module is bounded by n_0.

The Hochschild cohomology groups $\mathrm{H}^i(A)$, $i \geq 1$, of a finite dimensional algebra A, introduced in [8], have been much investigated lately (see, for instance, [5,11]). In this article, we shall study them for a class of algebras introduced and studied in [4], the so-called weakly shod algebras.

An algebra A is called *weakly shod* provided there exists a positive integer n_0 such that the length of any path in ind A from an injective to a projective module is bounded by n_0. It is not difficult to see that for a weakly shod algebra A all but finitely many indecomposable A-modules have its projective dimension at most one or its injective dimension at most one. Moreover, the class of weakly shod algebras includes the *shod algebras* [3] and the *quasitilted algebras* [7] (see Section 1 below for more details). A weakly shod algebra A is called *strict* provided it is not quasitilted. Our main result here is the following.

Theorem. *Let A be a strict weakly shod algebra. Then $\mathrm{H}^i(A) = 0$, for each $i \geq 2$.*

Observe that this result cannot be extended to arbitrary weakly shod algebras since there are quasitilted algebras with the second Hochschild cohomology different from zero (see, for instance [6]). The proof of our main result will be given in Section 2. Section 1 is devoted to some preliminary results while in Section 3 we characterize the strict weakly shod algebras with the first Hochschild cohomology equal to zero and give some examples.

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1. Preliminaries.

1.1. Throughout this article all algebras will be assumed to be (associative with unity) finite dimensional k-algebras, where k is an algebraically closed field. Given an algebra A, we will denote by $\operatorname{mod} A$ the category of all finitely generated left A-modules, while $\operatorname{ind} A$ denotes its full subcategory with one representative of each indecomposable A-module. By τ_A, we denote the Auslander-Reiten translate DTr on A and by Γ_A the Auslander-Reiten quiver of A. Let $X, Y \in \operatorname{ind} A$. A *path from X to Y in* $\operatorname{ind} A$ is a chain of nonzero morphisms

$$X = X_0 \xrightarrow{f_1} X_1 \xrightarrow{f_2} \cdots \xrightarrow{f_{t-1}} X_{t-1} \xrightarrow{f_t} X_t = Y \tag{1}$$

with $t > 0$, between indecomposable modules. We indicate the existence of a path from X to Y with the notation $X \rightsquigarrow Y$. If the morphisms f_i's in (1) are irreducible, we say that this path belongs to Γ_A. We say that X is a *predecessor* of Y and Y is a *successor* of X provided there is a path $X \rightsquigarrow Y$. Observe that each indecomposable module is a predecessor and a successor of itself. Let (1) $X_0 \longrightarrow X_1 \longrightarrow \cdots \longrightarrow X_t$ be a path of irreducible maps in Γ_A. If $\tau_A X_{j+1} = X_{j-1}$, for some $1 \le j \le t-1$, then we say that j is a *hook* in (1).

For unexplained notions on representations theory of algebras we refer the reader to [2].

1.2. The following result will be useful in our considerations. For a proof, we refer the reader to [2].

Proposition. *Let A be an algebra and $X \in \operatorname{ind} A$. Then*

(a) $\operatorname{pd}_A X \le 1$ *if, and only if,* $\operatorname{Hom}_A(I, \tau_A X) = 0$ *for each injective module I.*
(b) $\operatorname{id}_A X \le 1$ *if, and only if,* $\operatorname{Hom}_A(\tau_A^{-1} X, P) = 0$ *for each projective module P.*

1.3. Following [3], we say that an algebra A is *shod* provided for each indecomposable A-module X, its projective dimension $\operatorname{pd}_A X$ is at most one or its injective dimension $\operatorname{id}_A X$ is at most one. As observed in [7], a shod algebra has the global dimension at most 3. Also, a shod algebra of global dimension at most 2 is called *quasitilted* (see [7] for details) and if it has global dimension equal to 3 we shall call it a *strict shod algebra*. The main feature on shod algebras is the existence of a trisection in the category $\operatorname{ind} A$, which we shall now recall. For a given algebra A, denote by $\mathcal{L}_A$ and $\mathcal{R}_A$ the following two subcategories of $\operatorname{ind} A$:

$$\mathcal{L}_A = \{X \in \operatorname{ind} A\colon \text{ if } Y \rightsquigarrow X, \text{ then } \operatorname{pd}_A Y \le 1\}$$

$$\mathcal{R}_A = \{X \in \operatorname{ind} A\colon \text{ if } X \rightsquigarrow Y, \text{ then } \operatorname{id}_A Y \le 1\}.$$

We recall the following result from [3].

Theorem [3]**.** *The following are equivalent for an algebra A:*

(a) *A is shod;*
(b) $\mathcal{L}_A \cup \mathcal{R}_A = \operatorname{ind} A$*;*
(c) *Any path from an indecomposable injective module to an indecomposable projective module can be refined to a path of irreducible maps and any such*

refinement has at most two hooks, and, in case there are two, they are consecutive.

Moreover, if A satisfies one of the above conditions, then

$$\mathrm{Hom}_A(\mathcal{R}_A \setminus \mathcal{L}_A, \mathcal{L}_A) = 0 = \mathrm{Hom}_A(\mathcal{L}_A \cap \mathcal{R}_A, \mathcal{L}_A \setminus \mathcal{R}_A)$$

The existence of the trisection as above for quasitilted algebras has been established by Happel-Reiten-Smalø in [7].

1.4. As observed in [4], some of the results concerning, for instance, the structure of the Auslander-Reiten quiver of a shod algebra can be generalized by relaxing the condition (c) of the above theorem. With this in mind, we say that an algebra A is a *weakly shod algebra* provided there exists a positive integer n_0 such that the length of any path in $\mathrm{ind}\, A$ from an injective to a projective module is bounded by n_0, or equivalently, provided there exists a positive integer m_0 such that any path in $\mathrm{ind}\, A$ from an injective to a projective module pass through at most m_0 hooks (see [4] for details). It is not difficult then to see that a shod algebra is weakly shod (for $m_0 = 2$). Observe also that if A is weakly shod, then $\mathcal{L}_A \cup \mathcal{R}_A$ is cofinite in $\mathrm{ind}\, A$ (see [4]). Finally, we say that an algebra A is a *strict weakly shod algebra* provided it is weakly shod but it is not quasitilted.

1.5. An important step in our considerations is the possibility of writing a strict weakly shod algebra as an iteration of one-point extensions starting from tilted algebras. We shall now recall the precise statement. Let B be an algebra and $M \in \mathrm{mod}\, B$. We say that the algebra

$$B[M] = \begin{pmatrix} k & 0 \\ M & B \end{pmatrix}$$

is the *one-point extension of B by M*. The objects in $\mathrm{mod}\, B[M]$ can be written as triples (k^t, X, f) where $t \geq 0$, X is a B-module and $f\colon M^t \longrightarrow X$ is a morphism in $\mathrm{mod}\, B$. By taking $t = 0$ and $f = 0$, we can embed naturally the category $\mathrm{mod}\, B$ into $\mathrm{mod}\, B[M]$ (see, for instance, [2], for details). Observe, however, that the (unique) indecomposable projective $B[M]$-module which is not a B-module can be written as (k, M, Id_M), where Id_M is the identity map, and we shall refer to it as the *extended projective $B[M]$-module*.

1.6. For an algebra A, denote by $\mathcal{P}_A^f$ the set of the projective modules $P \in \mathrm{ind}\, A$ such that there exists a path $I \rightsquigarrow P$ where I is an indecomposable injective A-module. We define the following (partial) order in $\mathcal{P}_A^f$, (see [4]):

$$P \preceq Q \Leftrightarrow \exists \text{ a path } P \rightsquigarrow Q$$

We also recall the following result from [4] (see also [9]).

Theorem [4]**.** *Let A be a strict weakly shod algebra. Then, there are algebras $B = A_0, A_1, \ldots, A_t = A$ and A_i-modules M_i for each $i = 0, \ldots, t-1$ such that:*

(i) *B is a product of tilted algebras;*
(ii) *$A_{i+1} = A_i[M_i]$ for each $i = 0, \ldots, t-1$;*
(iii) *The extended projective A_{i+1}-module (k, M_i, Id_{M_i}) is a maximal element in $\mathcal{P}^f_{A_{i+1}}$ with the order defined above.*

1.7. For an algebra A, denote by $\mathrm{H}^i(A)$ its i-th Hochschild cohomology group (see [5, 8] for details). The next results, due to Happel, will be useful in our considerations. For a proof of them, we refer to [5].

Theorem [5]**.** *Let B be a connected tilted algebra of type Q. Then*
(i) $\mathrm{H}^0(B) = k$*;*
(ii) $\mathrm{H}^1(B) = 0$ *if, and only if, Q is a tree;*
(iii) $\mathrm{H}^i(B) = 0$ *for each $i \geq 2$.*

1.8. Theorem [5]**.** *Let $A = B[M]$. Then there exists a long exact sequence*

$$0 \longrightarrow \mathrm{H}^0(A) \longrightarrow \mathrm{H}^0(B) \longrightarrow (\mathrm{End}_A\, M)/k \longrightarrow \mathrm{H}^1(A) \longrightarrow \mathrm{H}^1(B) \longrightarrow$$
$$\longrightarrow \mathrm{Ext}^1_B(M, M) \longrightarrow \cdots \longrightarrow \mathrm{H}^i(A) \longrightarrow \mathrm{H}^i(B) \longrightarrow \mathrm{Ext}^i_B(M, M) \longrightarrow \cdots$$

2. The results.

2.1. Let A be a strict weakly shod algebra. The strategy of the proof of our main result will be to show that at each step in the iteration of one-point extension given in (1.6), the modules M_i satisfy $\mathrm{Ext}^j_{A_i}(M_i, M_i) = 0$ for $j > 0$ (using the notations of (1.6)) and then use Happel's long exact sequence given in (1.8). This will follow from the next two propositions.

2.2. Proposition. *Let $A = B[M]$ be a weakly shod algebra and assume that the extended projective A-module is a maximal element in $\mathcal{P}^f_A$. Then* $\mathrm{Ext}^1_B(M, M) = 0$.

Proof. Let N be an indecomposable direct summand of M. We shall first show that $\mathrm{Ext}^1_B(N, N) = 0$. Suppose this does not hold.

Since $\mathrm{Ext}^1_B(N, N) = \mathrm{D}\,\overline{\mathrm{Hom}}_B(N, \tau_B N)$, (see [2] for details), we then infer that $\mathrm{Hom}_B(N, \tau_B N) \neq 0$. It follows from [10] that

$$\tau_A(0, N, 0) = (\mathrm{Hom}_B(N, \tau_B N), \tau_B N, e_{\tau_B N}),$$

where $e_{\tau_B N}$ stands for the evaluation map from $\mathrm{Hom}_B(N, \tau_B N)$ to $\tau_B N$ (see [10] for details). Observe that

$$\mathrm{Hom}_A((k, M, Id_M), (\mathrm{Hom}_B(N, \tau_B N), \tau_B N, e_{\tau_B N})) \neq 0.$$

In particular, there exists a path from the extended projective A-module (k, M, Id_M) to $\tau_A(0, N, 0)$. Also, since N is an indecomposable summand of M, there exists a path (indeed a nonzero morphism) from $(0, N, 0)$ to (k, M, Id_M). Hence there is a path

$$(1) \qquad (k, M, Id_M) \longrightarrow \tau_A(0, N, 0) \longrightarrow (*) \longrightarrow (0, N, 0) \longrightarrow (k, M, Id_M).$$

Now, since (k, M, Id_M) is in $\mathcal{P}^f_A$, there exists an indecomposable injective A-module I and a path (2) in $\mathrm{ind}\, A$ from I to (k, M, Id_M). Glueing the paths (1) and (2), we get a path in $\mathrm{ind}\, A$ from an indecomposable injective A-module to an indecomposable projective A-module. Since B is weakly shod, we know from [4] that this path can be refined to a path of irreducible maps

$$I \longrightarrow \cdots \longrightarrow (k, M, Id_M) \longrightarrow \cdots \longrightarrow \tau_A(0, N, 0) \longrightarrow$$
$$\longrightarrow E \longrightarrow (0, N, 0) \longrightarrow (k, M, Id_M).$$

Observe that there exists a subpath in the above path which is a cycle in Γ_A through (k, M, Id_M). Using this latter path one can construct paths in Γ_A from I to (k, M, Id_M) with arbitrary length, a contradiction to the fact that A is a weakly shod algebra. Therefore, $\mathrm{Ext}^1_B(N, N) = 0$ for each indecomposable direct summand N of M. In particular, the result is proven if M is indecomposable. Suppose now that M is not indecomposable and that $\mathrm{Ext}^1_B(M, M) \neq 0$. So, there exists an indecomposable direct summand N_1 of M with $\mathrm{Ext}^1_B(M, N_1) \neq 0$. Write $M = N_1 \oplus N_2$ and observe that N_2 is not projective, since otherwise,

$$0 \neq \mathrm{Ext}^1_B(N_1 \oplus N_2, N_1) = \mathrm{Ext}^1_B(N_1, N_1)$$

which is a contradiction to the claim proven above. Consider now the indecomposable A-module $Z = (k, N_1, \pi_1)$ where $\pi_1 \colon N_1 \oplus N_2 \longrightarrow N_1$ is the canonical projection over N_1. Since $\mathrm{Ext}^1_B(M, N_1) \neq 0$, it follows from [7] that $\mathrm{id}_A\, Z \geq 2$. Now, by (1.2), there exists an indecomposable projective module P' such that $\mathrm{Hom}_A(\tau^{-1}Z, P') \neq 0$. Since there is a nonzero morphism $(k, M, Id_M) \longrightarrow Z$, we get a path

$$(k, M, Id_M) \longrightarrow Z \rightsquigarrow \tau_A^{-1} Z \longrightarrow P'$$

in ind A, a contradiction to the fact that the extended projective module (k, M, Id_M) is maximal in $\mathcal{P}^f_A$. Therefore, $\mathrm{Ext}^1_B(M, M) = 0$, as required. $\square$

2.3. Proposition. *Let $A = B[M]$ be a weakly shod algebra and assume that the extended projective A-module is a maximal element in $\mathcal{P}^f_A$. Then $\mathrm{Ext}^i_B(M, M) = 0$ for each $i \geq 2$.*

Proof. Suppose there exists an $i \geq 2$ such that $\mathrm{Ext}^i_B(M, M) \neq 0$. Then there exists an indecomposable summand M_1 of M such that $\mathrm{Ext}^i_B(M, M_1) \neq 0$. Clearly, then, $\mathrm{Ext}^i_A((0, M, 0), (0, M_1, 0)) \neq 0$. Denote by Z the quotient of the extended projective A-module (k, M, Id_M) by $(0, M_1, 0)$. Applying now $\mathrm{Hom}_A((0, M, 0), -)$ to the short exact sequence

$$0 \longrightarrow (0, M_1, 0) \longrightarrow (k, M, Id_M) \longrightarrow Z \longrightarrow 0$$

one gets, for each $j \geq 2$,

$$\cdots \longrightarrow \mathrm{Ext}^{j-1}_A((0, M, 0), Z) \longrightarrow \mathrm{Ext}^j_A((0, M, 0), (0, M_1, 0)) \longrightarrow$$
$$\longrightarrow \mathrm{Ext}^j_A((0, M, 0), (k, M, Id_M)) \longrightarrow \cdots$$

Observe that $\mathrm{id}_A(k, M, Id_M) \leq 1$. Indeed, if $\mathrm{id}_A(k, M, Id_M) \geq 2$, there would exist a nonzero morphism from $\tau^{-1}(k, M, Id_M)$ to a projective A-module (1.2) leading to a contradiction to the fact that (k, M, Id_M) is maximal in $\mathcal{P}^f_A$. Therefore, $\mathrm{Ext}^j_A((0, M, 0), (k, M, Id_M)) = 0$ for each $j \geq 2$. Since $\mathrm{Ext}^i_A((0, M, 0), (0, M_1, 0))$

$\neq 0$, we then infer that $\mathrm{Ext}_A^{i-1}((0,M,0),Z) \neq 0$. Consequently, in case $i = 2$, $\mathrm{Hom}_A(Z, \tau_A(0,M,0)) \neq 0$ (recall that $\mathrm{Ext}_A^1((0,M,0),Z) = \mathrm{D}\,\overline{\mathrm{Hom}}_A(Z, \tau_A(0,M,0))$, see [2]). In particular, there exists an indecomposable direct summand N of M such that $\mathrm{Hom}_A(Z, \tau_A(0,N,0)) \neq 0$. We obtain then a path

$$(k, M, Id_M) \longrightarrow Z \longrightarrow \tau_A(0,N,0) \rightsquigarrow (0,N,0) \longrightarrow (k, M, Id_M)$$

in Γ_A. Since $(k, M, Id_M) \in \mathcal{P}_A^f$ and using the same argument in the proof of Proposition 2.2, one can get paths in Γ_A from an indecomposable injective to an indecomposable projective module with arbitrary length, a contradiction to our hypothesis on A being weakly shod.

Now, in case $i \geq 3$, we infer that $\mathrm{id}_A Z \geq 2$ and then we get a path

$$(k, M, Id_M) \longrightarrow Z \rightsquigarrow \tau_A^{-1} Z \longrightarrow P'$$

with P' be a projective module (1.2), a contradiction to the fact that (k, M, Id_M) is a maximal element in $\mathcal{P}_A^f$. $\square$

2.4. We can now prove our main result.

Theorem. *Let A be a strict weakly shod algebra. Then $\mathrm{H}^i(A) = 0$, for each $i \geq 2$.*

Proof. Let A be a strict weakly shod algebra. So, by (1.6), there are algebras $B = A_0, A_1, \ldots, A_t = A$ and A_i-modules M_i for each $i = 0, \ldots, t-1$ such that: (i) B is a product of tilted algebras; (ii) $A_{i+1} = A_i[M_i]$ for each $i = 0, \ldots, t-1$; and (iii) the extended projective A_{i+1}-module (k, M_i, Id_{M_i}) is a maximal element in $\mathcal{P}_{A_{i+1}}^f$. We shall use induction on $t \geq 1$ to get our result. Suppose $t = 1$, that is, $A = B[M]$, where B is a product of tilted algebras and the extended indecomposable projective A-module is maximal in $\mathcal{P}_A^f$. Then, by (1.7), $\mathrm{H}^i(B) = 0$, for each $i \geq 2$. Since $\mathrm{Ext}_A^i(M,M) = 0$ for each $i \geq 1$, we get from Happel's long exact sequence that $\mathrm{H}^i(A) = 0$ for each $i \geq 2$. The above argument can be indeed made at each step of the iteration of one-point extensions described in (1.6) in order to get the desired result. The proof of our main theorem is now completed. $\square$

3. The first Hochschild cohomology of a strict weakly shod algebra.

3.1. We have seen that the higher Hochschild cohomology groups for a strict weakly shod algebra A vanish. However, $\mathrm{H}^1(A)$ will clearly depend on the types of the tilted algebras which are components of B and properties of the modules M_i (using the notations of (1.6)). In order to give our next result, we shall recall some notions.

3.2. Let A be a triangular algebra and let x a vertex in the ordinary quiver Q_A of A and denote by A^x the full subcategory of A generated by the non-predecessors of x in Q_A. We say that x is *separating* provided the restricitions to A^x of rad P_x is separated as an A^x-module, that is, for each connected component C of A^x, the restrictions of rad P_x to C is either zero or indecomposable. We also recall the following useful result (see [1]).

Lemma. *Let $A = B[M]$ and let x be the vertex of Q_A corresponding to the extended projective A-module. Then, the morphism $\mathrm{H}^1(A) \longrightarrow \mathrm{H}^1(B)$ of (1.8) is injective if, and only if, x is separating and M is the direct sum of pairwise orthogonal bricks.*

3.3. The next result will then follow easily from the above together with our considerations along the paper.

Proposition. *Let A be a strict weakly shod algebra. Using the notations of (1.6),* $\mathrm{H}^1(A) \cong \mathrm{H}^1(B)$ *if, and only if, for each $i \geq 0$,*

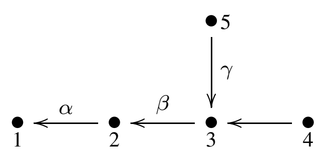
(a) *the extended projective A_{i+1}-module is separating; and*
(b) *the module M_i is a direct sum of pairwise orthogonal bricks.*

Corollary. *Let A be a strict weakly shod algebra. Using the notations of (1.6),* $\mathrm{H}^1(A) = 0$ *if, and only if,*

(a) *B is a product of connected tilted algebras of tree type;*
(b) *for each $i \geq 0$, the extended projective A_{i+1}-module is separating; and*
(c) *for each $i \geq 0$, the module M_i is a direct sum of pairwise orthogonal bricks.*

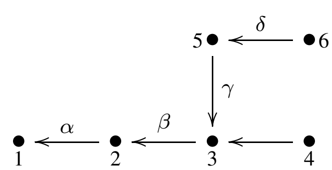
3.4. We shall finish our article by exhibiting some examples.

Examples. (a) Let B be the k-algebra given by the quiver:



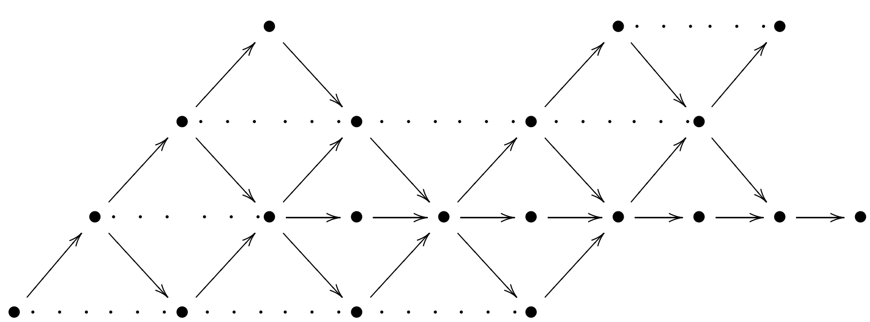
with $\alpha\beta\gamma = 0$.

It is not difficult to see that B is a tilted algebra of type $\mathbf{D}_5$. Therefore, by (1.7), $\mathrm{H}^1(B) = 0$. Consider $M = \tau^{-2}P_3$, that is, the indecomposable B-module of dimension vector $\underline{\dim}\, M = (0,0,1,0,1)$ and $A = B[M]$. Then A is the k-algebra given by the quiver



with $\alpha\beta\gamma = \beta\gamma\delta = 0$

and its Auslander-Reiten quiver is



Clearly, A is a strict shod algebra, and since M is a brick and the extended projective A-module is separating we infer that $\mathrm{H}^1(A) = 0$.

(b) Let B be the k-algebra given by the quiver

$$\underset{1}{\circ} \xleftarrow{\ \alpha\ } \underset{2}{\circ} \xleftarrow{\ \beta\ } \underset{3}{\circ} \qquad\qquad \text{with } \alpha\beta = 0\,.$$

The algebra B is tilted of type $\tilde{\mathbf{A}}_3$ (with a complete slice in its preinjective component) and therefore by (1.7), $\mathrm{H}^1(B) \neq 0$. Consider the one-point extension $A = B[S_3]$ of B by the simple B-module S_3 associated to the vertex 3 which is indeed the unique indecomposable B-module of projective dimension 2. It is not difficult to see that there are then only two indecomposable A-modules which have projective dimension greater than 2, namely, S_3 and S_4. Since $\mathrm{pd}_A\, S_3 = 2$, $\mathrm{pd}_A\, S_4 = 3$, $\mathrm{id}_A\, S_3 = 1$, and $\mathrm{id}_A\, S_4 = 0$, we infer that A is a strict shod algebra. Also, it follows from the above considerations that $\mathrm{H}^1(A) \neq 0$.

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Résumé substantiel en français. Les groupes de cohomologie de Hochschild, $H^i(A)$, $i \geq 1$, d'une algèbre de dimension finie A, introduite en [6], ont récemment été l'objet de plusieurs travaux. Dans cet article, nous les étudions pour une classe d'algèbres, introduite et étudiée en [7], que nous appelons *algèbres faiblement chaussées*.

Une algèbre A est dite *faiblement chaussée* s'il existe un entier positif n_0 tel que la longueur de chaque chemin de non isomorphismes non nuls entre A-modules indécomposables d'un A-module injectif vers un A-module projectif est bornée par n_0. Il n'est pas difficile de vérifier que si A est faiblement chaussée, alors tous les A-modules indécomposables sauf un nombre fini de classes d'isomorphisme ont une dimension projective ou une dimension injective au plus égale à un. Par conséquent, la classe des algèbres faiblement chaussée contient celle des algèbres chaussées, et celle des algèbres quasi-inclinées. Une algèbre faiblement chaussée est dite *stricte* si elle n'est pas quasi-inclinée. Notre résultat principal est le suivant.

Théorème. *Soit A une algèbre faiblement chaussée stricte. Alors $H^i(A) = 0$ pour chaque $i \geq 2$.*

Ce résultat ne peut être généralisé aux algèbres faiblement chaussées arbitraires, certaines algèbres quasi-inclinées ayant leur dimension groupe de cohomologie de Hochschild différente de zéro.

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