Ann. Sci. Math. Québec **25** (2001), no. 2, 199–209.

ON THE TAMENESS OF TRIVIAL EXTENSIONS OF MONOMIAL ALGEBRAS

C. NOVOA AND J. A. DE LA PEÑA

RÉSUMÉ. Soit k un corps algébriquement clos et A une k-algèbre de dimension finie. Nous supposons que $A = kQ/I$, où Q est un carquois sans cycles orientés et I est engendrée par des chemins de Q. Nous considérons l'extension triviale $T(A)$ et donnons des conditions sur A de sorte que $T(A)$ soit docile. En particulier, nous montrons que $T(A)$ est docile si, et seulement si, A est dérivablement docile.

ABSTRACT. Let k be an algebraically closed field and A a finite dimensional k-algebra. We assume that $A = kQ/I$ where Q is a quiver without oriented cycles and I is generated by paths in Q. We consider the trivial extension $T(A)$ of A and give conditions on A in order that $T(A)$ is tame. In particular we show that $T(A)$ is tame if, and only if, A is derived-tame.

Let k be an algebraically closed field and A be a finite dimensional k-algebra (associative, with identity) which moreover we assume to be basic and connected, hence of the form $A = kQ/I$ where Q is a finite connected quiver and I is an ideal of the path algebra kQ (see [10]). We denote by $\operatorname{mod} A$ the category of finite dimensional left A-modules, and by $D = \operatorname{Hom}_k(-, k)$ the standard duality between $\operatorname{mod} A$ and $\operatorname{mod} A^{op}$.

The *trivial extension* $T(A) = A \ltimes DA$ of A by its minimal injective cogenerator bimodule ${}_ADA_A$, has as additive structure that of the group $A \oplus DA$ and multiplication defined by

$$(a, f)(b, g) = (ab, ag + fb)$$

for $a, b \in A$ and $f, g \in DA$. Trivial extensions are self-injective algebras which have played an important role in the representation theory of algebras. The problem of studying the representation type of $T(A)$ has been considered in the work of several authors (see [2, 21, 22, 24]). For example, in [2] it is shown that $T(A)$ is representation-finite if, and only if, A is an iterated tilted algebra of a hereditary algebra $k\Delta$ where Δ is of Dynkin type. In this paper, we characterize those algebras of the form $A = kQ/I$, where Q has no oriented cycles (that is, A is triangular) and I is generated by a set of paths in Q (that is, A is monomial), for which $T(A)$ is tame. We recall here that an algebra B is tame if for each dimension n, the indecomposable B-modules

Reçu le 27 mars 2001 et, sous forme définitive, le 19 septembre 2001.

of dimension n may be described by a finite number of one-parametric families of modules. Our results generalize previous statements in the literature [13, 26].

Let $A = kQ/I$ be as above. If Q is a tree, then $T(A)$ is tame if, and only if, A is derived equivalent to a hereditary algebra of Dynkin or Euclidean type, to a tubular algebra or to an algebra of type $S(n,m)$ – see also [13]. In case Q is not a tree, we shall use covering techniques as introduced in [11, 20] to show that $T(A)$ is tame if, and only if, A is derived equivalent to a deformation of a *skewed-gentle* algebra as defined in [15]. Moreover, in any of these cases, $T(A)$ is tame if, and only if, A is derived-tame.

1. Fundamental concepts.

1.1. Let A be a finite dimensional k-algebra of the form $A = kQ/I$. We denote by e_x the primitive idempotent corresponding to a vertex x in Q, hence $1_A = \sum_{x\in Q_0} e_x$, where Q_0 is the set of vertices of Q. Following [10], we shall sometimes consider the algebra A as a locally bounded k-category with $A(x,y) = e_y A e_x$ for any $x, y \in Q_0$.

Given an A-module M, the *one-point extension* $A[M]$ is defined as the k-algebra

$$\begin{pmatrix} A & M \\ 0 & k \end{pmatrix}$$

with the usual matrix operations. For a sink $x \in Q_0$, the *reflection* $S_x^+ A$ of A at x is the quotient of the extension $A[I_x]$ by the two-sided ideal generated by e_x, where I_x denotes the injective envelope of the simple A-module S_x associated with the vertex x. Dually, for a source $y \in Q_0$, $S_y^- A$ is the quotient of the coextension $[P_y]A$ by the two-sided ideal generated by e_y, where P_y is the projective cover of S_y.

For tilting theory, tubular algebras and other concepts, we refer the reader to [25].

1.2. The *repetitive category* $\hat{A}$ of A is the self-injective locally finite dimensional matrix algebra defined by

$$\hat{A} = \begin{bmatrix} \ddots & & & & 0 \\ \ddots & A & & & \\ & DA & A & & \\ & & DA & A & \\ 0 & & & \ddots & \ddots \end{bmatrix}$$

where matrices have only finitely many non-zero coefficients. Clearly, there is an automorphism $\nu_A\colon \hat{A} \to \hat{A}$ such that the orbit space $\hat{A}/(\nu_A)$ is isomorphic to the trivial extension $T(A)$. Hence, the orbit map $\hat{A} \to T(A)$ is a Galois covering defined by the action of the cyclic group generated by ν_A (isomorphic to $\mathbb{Z}$), and the category mod $\hat{A}$ is equivalent to the $\mathbb{Z}$-graded $T(A)$-modules. We shall consider the *push-down functor* $F_\lambda\colon \operatorname{mod}\hat{A} \to \operatorname{mod} T(A)$ as defined in [11].

Observe that for a source $x \in Q_0$, there is an isomorphism $\widehat{S_x^- A} \cong \hat{A}$. Moreover, in [17] it is shown that for triangular algebras A and B, we have $\hat{A} \cong \hat{B}$ if, and only if, there is an admissible sequence of vertices $x_1, x_2, \ldots, x_t$ such that $B = S_{x_t}^- \ldots S_{x_2}^- S_{x_1}^- A$.

1.3. Recall that an algebra A is said to be *tame* if for every dimension $d \in \mathbb{N}$, the indecomposable A-modules of dimension d may be parametrized by a finite number $\nu(d)$ of families $M_i \otimes_{k[t]} S_\lambda$ (with $\lambda \in k$) where M_i is an $A - k[t]$-bimodule, finitely generated free as right $k[t]$-module, $1 \leq i \leq \nu(d)$. If A is tame, then A is domestic (resp. of polynomial growth) if $\nu(d) \leq$ constant (resp. $\nu(d) \leq d^m$ for some $m \in \mathbb{N}$).

By [9], given a Galois covering $F\colon B \to A$ defined by the action of a group G, if A is tame (resp. domestic, polynomial growth), then so B is. The converse may fail [14].

In [5], it is shown that the repetitive category $\hat{A}$ is tame and the push-down functor $F_\lambda\colon \mathrm{mod}\,\hat{A} \to \mathrm{mod}\,T(A)$ is dense if, and only if, A is tilting-cotilting equivalent to a hereditary algebra of Dynkin or Euclidean type or a tubular algebra. It follows that $T(A)$ is tame in these cases.

1.4. For a triangular algebra A, the stable module category $\underline{\mathrm{mod}}\hat{A}$ is equivalent as triangulated category to the *derived category* $D^b(A)$ of the module category mod A, [16]. Further, by [27], if A and B are tilting-cotilting equivalent, then $\underline{\mathrm{mod}}\hat{A} \simeq \underline{\mathrm{mod}}\hat{B}$ (as triangulated categories). It is shown in [3] that this implies that $T(A)$ is tame if, and only if, $T(B)$ is tame. Moreover, $\hat{A}$ is tame if, and only if, so $\hat{B}$ is.

Following [23], we say that A is *derived-tame* if gℓ dim $A < \infty$ and $\hat{A}$ is tame. By [19] or [23], if A is derived-tame and derived equivalent to B, then B is derived-tame. We give some examples:

(a) If A is hereditary of Dynkin or Euclidean type or a tubular algebra, then A is derived-tame.

(b) Let $A = S(n,m)$ be the (semichain) poset algebra given by the quiver:

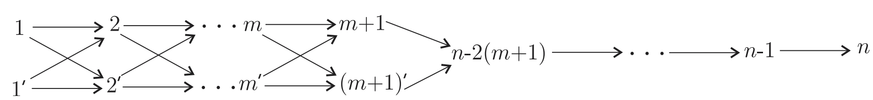

Figure 1.

As observed in [23], A is derived-tame. The trivial extension $T(A)$ is Morita equivalent to a quotient of the "clannish" algebra (see [8]):

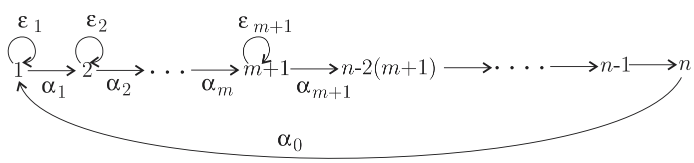

Figure 2.

bounded by $\varepsilon_i^2 = \varepsilon_i$, $\alpha_{i-1}\alpha_i = 0$ for $i = 1, \ldots, m+1$. Therefore $T(A)$ is tame.

(c) Let $A = kQ/I$, where Q is the quiver

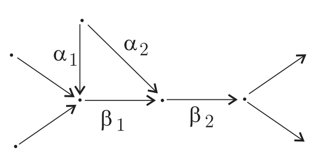

Figure 3.

and I is generated by $\alpha_1\beta_1$ and $\alpha_2\beta_2$. Then $T(A)$ is tame but A is not derived equivalent to any of the examples given in (a) and (b).

For A as in the above examples, the *Euler form* χ_A of A is non-negative. Recall that

χ_A is defined on the Grothendieck group $K_0(A)$ as the quadratic form satisfying

$$\chi_A([M]) = \sum_{i=0}^{\infty}(-1)^i \dim_k \operatorname{Ext}_A^i(M, M),$$

for any module M.

1.5. Let $A = kQ/I$ be a k-category. Then A is said to be *special biserial* if the following holds:

(B1) At every vertex of Q at most two arrows start and at most two stop;
(B2) For every arrow β in Q, there is at most one arrow α with $\alpha\beta \notin I$ and at most one arrow γ with $\beta\gamma \notin I$.

A special biserial category A is said to be *gentle* if moreover:

(B3) The set I is generated by monomial relations of length two;
(B4) For every arrow β, there is at most one arrow α' with $\alpha'\beta \in I$ and at most one arrow γ' with $\beta\gamma' \in I$.

We have the following result.

Proposition. [4, 21, 24, 26]. *Let A be a triangular algebra. Then $\hat{A}$ is special biserial if, and only if, A is gentle. In this case, $\hat{A}$ is of polynomial growth (resp. domestic) if, and only if, $T(A)$ is of polynomial growth (resp. domestic). Moreover, $T(A)$ is of polynomial growth if, and only if, A contains at most one cycle.*

1.6. Let $A = kQ/I$ be a triangular monomial algebra. Then the *universal Galois covering* $F\colon U_A \to A$ is defined by the action of a free group G acting on the tree category U_A. Let $r(G)$ be the rank of the group G, hence Q is a tree if, and only if, $r(G) = 0$.

Clearly, we may define a functor $\hat{F}\colon \hat{U}_A \to \hat{A}$, $(u, n) \mapsto (Fu, n)$ in such a way that G acts on $\hat{U}_A$ with $\hat{F}g = \hat{F}$ for every $g \in G$. Then, there is a covering functor $T(F)\colon T(U_A) \to T(A)$ invariant under the action of G and such that the following diagram commutes

$$\begin{array}{ccc} \hat{U}_A & \xrightarrow{\nu_{U_A}} & T(U_A) \\ \hat{F}\downarrow & & \downarrow T(F) \\ \hat{A} & \xrightarrow[\nu_A]{} & T(A) \end{array}$$

In particular, if $T(A)$ is tame, then $\hat{A}$, $T(U_A)$ and $\hat{U}_A$ are also tame.

1.7. Finally, we recall some concepts from [15]. Let Q be a quiver with a subset L of loops in Q. The arrows in Q but not in L are called *ordinary arrows*. Let I be an ideal of kQ which includes $\varepsilon^2 - \varepsilon \in I$ for every $\varepsilon \in L$. Then $A = kQ/I$ is a *skewed-gentle* algebra if it satisfies:

(C1) At any vertex of Q there start at most two arrows and at most two end.
(C2) For every ordinay arrow β, there is at most one arrow α with $\alpha\beta \notin I$ and at most one arrow γ with $\beta\gamma \notin I$.

(C3) The ideal I is generated by paths of length two.

(C4) For every ordinary arrow β, there is at most one arrow α' with $\alpha'\beta \in I$ and at most one arrow γ' with $\beta\gamma' \in I$.

Proposition. [15, (4.9)]. *Let A be an algebra derived equivalent to a skewed-gentle algebra. Then $T(A)$ is tame.*

Sketch of proof. Let B be a skewed-gentle algebra such that A and B are derived equivalent. Then $\hat{A}$ and $\hat{B}$ are stably equivalent, which implies by [17] that $T(A)$ and $T(B)$ are stably equivalent. Recall from [15] that $T(B)$ is tame (in fact, quasi-clannish in the notation of [15]). By [19] or [23], $T(A)$ is also tame. □

2. The main result.

2.1. We state the main theorem of our work.

Theorem. *Let A be a triangular monomial algebra. Consider the universal covering $F\colon U_A \to A$ defined by the action of the free group G. The following are equivalent:*

(1) $T(A)$ *is tame*

(2) A *is derived-tame*

(3) *One of the following holds for U_A:*

(i) $r(G) = 0$ *and* $U_A = A$ *is derived equivalent to a hereditary algebra of Dynkin or Euclidean type or a tubular algebra or a semichain* $S(n, m)$*;*

(ii) $r(G) > 0$ *and every convex subcategory B of U_A is derived equivalent to a hereditary algebra of type* $\mathbb{A}_n$*;*

(iii) $r(G) > 0$ *and there are convex subcategories B_n of U_A such that* $B_n \subset B_{n+1}$, $\lim_{n\to\infty} B_n = U_A$ *and B_n is derived equivalent to a semichain* $S(n, c(n))$ *with* $\lim_{n\to\infty} c(n) = \infty$.

(4) *One of the following holds for A:*

(i) $r(G) = 0$ *and A is derived equivalent to a hereditary algebra of Dynkin or Euclidean type or a tubular algebra or a semichain* $S(n, m)$*;*

(ii) $r(G) > 0$ *and A is tilting-cotilting equivalent to a deformation of a skewed-gentle algebra.*

Recall that an algebra A_1 is said to be a *deformation* of A_0 if A_0 lies in the Zariski closure of the $GL_d(k)$-orbit of A_1 in the variety alg (d) of k-algebras of dimension d.

2.2. We shall proof the implication (3) ⇒ (4) in Section (2.6). In the rest of this section we complete the demonstration of other implications.

(1) ⇒ (2) was observed in (1.4).

(2) ⇒ (3): Assume that $\hat{A}$ is tame. If $r(G) = 0$, A is a tree algebra and Geiss [13] has shown that A is derived equivalent to one of the 4 types of algebras stated.

Suppose that $r(G) > 0$. By (1.6), U_A is a tree category such that $\hat{U}_A$ is tame. Hence condition (2) holds for U_A. Moreover by [13], the Euler form χ_B of any finite subcategory B of U_A is non-negative and by [6], U_A does not accept any convex subcategory derived equivalent to $\mathbb{E}_p$ ($p = 6, 7, 8$) or to a tubular algebra (since otherwise, U_A would be finite).

Let B be a convex subcategory of U_A not derived equivalent to a hereditary algebra of type $\mathbb{A}_n$, then B is derived equivalent to a hereditary algebra of type $\mathbb{D}_n$ or to a semichain $S(n,c)$ (because $\chi_B \geq 0$). Assume that B is derived equivalent to an algebra of type $\mathbb{D}_n$, then take $1 \neq g \in G$ and B_1 the connected closure of B and $g(B)$ in U_A. By the description of the algebras derived equivalent to $\mathbb{D}_n$ (as given for example in [18]), B_1 is not of Dynkin type $\mathbb{D}_n$. Hence B_1 is of type $S(n_1, c_1)$ with $c_1 \geq 1$. Let B_2 be a connected convex subcategory of U_A containing B_1 and $g(B_1)$, which clearly is of type $S(n_2, c_2)$ with $c_2 > c_1$. Choosing an increasing sequence $(B_n)_n$ of convex subcategories of U_A with $\lim_{n\to\infty} B_n = U_A$, it is clear that U_A is of type (iii).

(4) $\Rightarrow$ (1): The case $r(G) = 0$ was observed in (1.4). Suppose $r(G) > 0$ and A is tilting-cotilting equivalent to an algebra B_1 which is a deformation of a skewed-gentle algebra B_0. Take a family of algebras $(B_\lambda)_{\lambda\in k}$ such that $B_\lambda \cong B_1$ for $\lambda \neq 0$. By [1], $T(A)$ is derived equivalent to $T(B_1) \cong T(B_\lambda)$ for $\lambda \neq 0$ and clearly $\lim_{\lambda\to 0} T(B_\lambda) = T(B_0)$. Then $T(A)$ is a deformation of the algebra $T(B_0)$ which is tame by (1.7). By [12], $T(A)$ is tame. $\square$

2.3. Let $F\colon U_A \to A$ be the universal covering of A defined by the action of the free group G of rank $r(G) > 0$. Assume moreover that every convex subcategory B of U_A is derived equivalent to a semichain $S(n,m)$. Then, we know that U_A does not accept convex subcategories which are derived equivalent to $\mathbb{E}_p$ $(p = 6,7,8)$ or to a tubular algebra and moreover $\chi_{U_A} \geq 0$. The next Proposition follows from the main technical result in [7].

Proposition. *Under the above hypothesis, U_A is tilting-cotilting equivalent to a semi-tree $T\{E\}$.*

We recall that $T\{E\}$ is a *semi-tree* if the following holds:

(D0) $T = kQ'/J$ is a tree algebra and E is a set of vertices of Q'_0.

(D1) At each vertex of E starts at most one arrow and at each vertex of E stops at most one arrow.

(D2) The ideal J is generated by paths of length 2 and 3.

(D3) If $a \xrightarrow{\alpha} b \xrightarrow{\beta} c$ is an element of J, then $b \notin E$. Moreover, all other relations in J containing α stop at the vertex b, and those containing β start at b.

(D4) The generators of J of length 3 have the form $\varepsilon\colon a \longrightarrow a' \xrightarrow{\alpha} b \xrightarrow{\beta} c'$ with c' an end vertex of Q' or dually $\varepsilon'\colon a' \xrightarrow{\alpha} b \xrightarrow{\beta} c' \longrightarrow c$ with a' an end vertex or they come as a pair ($\varepsilon\colon a \longrightarrow a' \xrightarrow{\alpha} b \xrightarrow{\beta} c', \varepsilon'\colon a' \xrightarrow{\alpha} b \xrightarrow{\beta} c' \longrightarrow c$). In each case, the vertices a', b and c' do not belong to E, no other generator of J contains one of the arrows α or β, and in a' and c' do not start or stop any other arrow.

(D5) Each convex hereditary subcategory of T is of type $\mathbb{A}_n$.

Then, by definition $T\{E\} = k\bar{Q}/\bar{I}$, where $\bar{Q}_0$ are those vertices of Q' not in E and e^+, e^- for every $e \in E$; the arrows of $\bar{Q}$ are $t \to t'$ if $t, t' \notin E$, for every $e \to t$ with $e \in E$, two arrows $e^+ \to t$, $e^- \to t$ and dually for $t \to e$ with $e \in E$. Finally, $\bar{I} = \pi^{-1}(J)$ for the natural epimorphism $\pi\colon k\bar{Q} \to kQ'$.

In the Proposition, we may choose T and E in such a way that there is a free action of G on T and on E satisfying that $g\psi = \psi g$, where $\psi\colon\ D^b(U_A) \xrightarrow{\sim} D^b(T\{E\})$ is the triangular equivalence induced by the tilting-cotilting sequence.

Proof. From [7], we may find T and E satisfying (D1)-(D5). Indeed, observe that every step in the proof is carried by means of either (APR)-tilts or reflections S_x^+ and S_y^-, hence all steps remain in the domain of tilting-cotilting equivalences.

Choose E a maximal possible set satisfying (D1)-(D5). By definition G acts on E and hence on T. Clearly the action is compatible with the (APR)-tilts and reflections. □

Corollary. *Let $F\colon U_A \to A$ be a covering as indicated above. Then there is a pair $(\bar{T}, \bar{E})$ satisfying (D1) to (D4) and*

(D5') *Each convex hereditary subcategory of $\bar{T}$ is of type $\mathbb{A}_n$ or $\tilde{\mathbb{A}}_n$, and such that A is tilting-cotilting equivalent to $\bar{T}\{\bar{E}\}$.*

Proof. Choose the pair (T, E) as in the Proposition. Clearly we may define $\bar{T} = T/G$ and $\bar{E} = E/G$ satisfying (D1)-(D4) and (D5'). Since G is a free group, acting freely on T, then results of Asashiba [1] imply that A is tilting-cotilting equivalent to $T\{E\}/G = \bar{T}\{\bar{E}\}$. □

2.4. Lemma. *Let $B = C\{E\}$ be a finite dimensional algebra, where the pair (C, E) satisfies (D1) to (D4) and (D5'). Then B is a deformation of B_0 such that $B_0 = C_0\{E_0\}$ and C_0 is a gentle algebra.*

Proof. Observe that in case C does not contain relations of length 3, then C is gentle. Suppose $\varepsilon\colon a' \xrightarrow{\alpha} b \xrightarrow{\beta} c' \xrightarrow{\gamma} c$ is a relation in C with a' an end vertex. By (D4), the module $M = \operatorname{rad} P_{a'}$ is indecomposable and there is an exact sequence

$$0 \to N \to M \to S_b \to 0.$$

The algebra C is the one-point extension $C_1 = C'[M]$ and we may define algebras $C_\lambda = C'[M_\lambda]$, for $\lambda \in k$, where $M_\lambda\colon C' \to \operatorname{mod} k$ is the representation

$$M_\lambda(\delta) = \begin{bmatrix} N(\delta) & 0 \\ \lambda f_\delta & S_b(\delta) \end{bmatrix}, \qquad \text{where} \quad M(\delta) = \begin{bmatrix} N(\delta) & 0 \\ f_\delta & S_b(\delta) \end{bmatrix}.$$

The algebra $C_0 = C'[N \oplus S_b]$ is a degeneration of $C_1 = C$ which has the quiver of C_1 plus an additional arrow $a' \xrightarrow{\theta} c'$ and instead of the relation ε we have two relations $\alpha\beta$ and $\theta\gamma$.

Since by (D4), $b, c' \notin E$, then the pair (C_0, E) satisfies (D1) to (D4) and (D5'). Hence $B_0 = C_0\{E\}$ is a degeneration of $B = C\{E\}$. Proceeding inductively along all relations of length 3 in B, we get an algebra B' degeneration of B such that $B' = C'\{E'\}$ with the pair (C', E') satisfying (D1) to (D4) and (D5') but no relations in C' of length 3. Hence C' is a gentle algebra. □

2.5. Lemma. *Let C be a gentle algebra and E a set of vertices of C such that (C, E) satisfies (D1), (D2), (D3) and (D5'). Then $C\{E\}$ is Morita equivalent to a skewed-gentle algebra.*

Proof. Define $\bar{C}$ an algebra obtained from C by adding a loop ε_e at $e \in E$. Moreover, $\varepsilon_e^2 = \varepsilon_e$ in $\bar{C}$ and whenever $a \xrightarrow{\alpha} b \xrightarrow{\beta} c$ in C with $b \in E$, then $\alpha\beta = 0$ in $\bar{C}$.

We show that $C\{E\}$ and $\bar{C}$ are Morita equivalent. Since C is gentle, up to duality, a vertex $e \in E$ appears in one of the ways illustrated by Figure 4. Then $C\{e\}$ is Morita equivalent to the algebras of Figure 5 and $\varepsilon_e^2 = \varepsilon_e$ in every case.

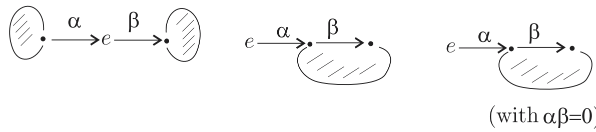

Figure 4.

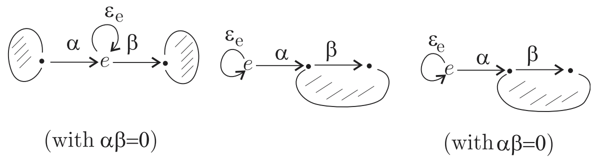

Figure 5.

Finally, to check that $\bar{C}$ is skewed-gentle is routine. □

2.6. Proof of (3) ⇒ (4) **of** (2.1)**.** Let $F\colon U_A \to A$ be a Galois covering defined by the action of the free group G. Without loss of generality, we may assume that $r(G) > 0$.

In case every convex subcategory of U_A is derived equivalent to a hereditary algebra of type $\mathbb{A}_n$, then U_A (and hence A) is a gentle category.

In the remaining case, we are in the situation considered in (2.3). Therefore by (2.3) and (2.4), then A is tilting-cotilting equivalent to an algebra which is a deformation of $A_0 = C_0\{E_0\}$ where C_0 is a gentle algebra and the pair (C_0, E_0) satisfies (D1), (D2), (D3) and (D5'). By (2.5), A_0 is a skewed-gentle algebra. □

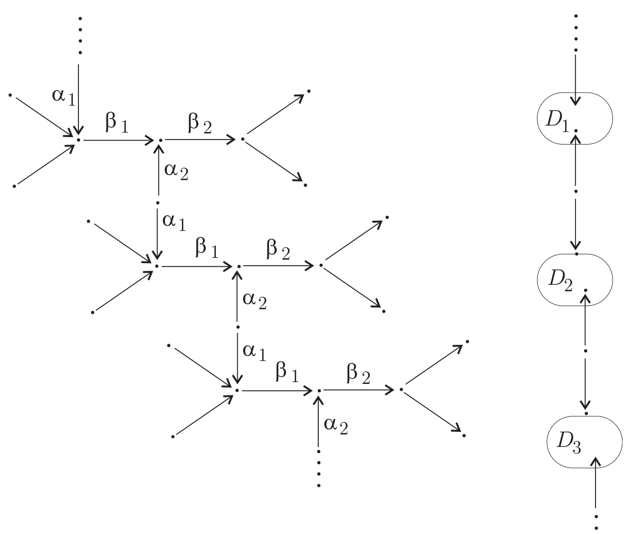

Figure 6.

3. Some examples and remarks.

3.1. The algebra A in example (1.4.c) accepts a universal covering $U_A = k\tilde{Q}/\tilde{I}$ defined by the action of $\mathbb{Z}$, where $\tilde{Q}$ is given as in Figure 6 and $\tilde{I}$ is generated by all relations of the form $\alpha_1\beta_1$, $\alpha_2\beta_2$. Consider the schematic representation of U_A, where each D_i

denotes a hereditary algebra of type $\tilde{\mathbb{D}}_6$. The full subcategory B of U_A formed as the connected hull of $D_1, D_2, \ldots, D_m$ is tilting-cotilting equivalent to $S(8m-1, 2m-1)$, showing that U_A is of type (3,iii) in the theorem.

Moreover, as shown in (2.4), the algebra A is a degeneration of $B = kQ'/I'$ where Q' is the quiver of Figure 7 and I' is generated by $\alpha\beta_1\beta_2$. The algebra B is of type (4,ii) in the Theorem.

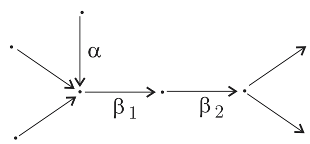

Figure 7.

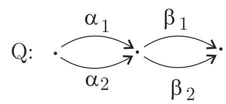

Figure 8.

3.2. For a locally bounded category A with possibly infinitely many objects, we say that the Euler form χ_A is non-negative (write $\chi_A \geq 0$) if for every full convex finite subcategory B of A we have $\chi_B \geq 0$. Using [13] and arguments similar to those in (2.2) we readily obtain:

Corollary. *Let A be a triangular monomial algebra. Then $T(A)$ is tame if, and only if, $\chi_{U_A} \geq 0$.*

3.3. It is not true that for a triangular algebra A, the condition $\chi_A \geq 0$ implies that A is tame. Consider $A = kQ/I$ given by the quiver of Figure 8 and I generated by $\alpha_1\beta_1 - \alpha_2\beta_2$ and $\alpha_1\beta_2$. As shown in [14], if char $k = 2$, then A is wild and

$$\chi_A(a, b, c) = (a - b + c)^2.$$

On the other hand, the algebra $A' = kQ/I'$ with I' generated by $\alpha_1\beta_1$ and $\alpha_2\beta_2$ accepts a covering $U_{A'} \to A'$ defined by the action of a free group in two generators. The category $U_{A'}$ is of type (3,ii) in the Theorem.

Acknowledgments This paper was done during a postdoctoral stay of the first named author at UNAM. Both authors thankfully acknowledge the finantial support of UNAM and CONACyT, México.

Résumé substantiel en français. Soit k un corps algébriquement clos et A une k-algèbre de dimension finie. Nous supposons $A = kQ/I$ avec Q un carquois sans cycles orientés (on dit alors que A est triangulaire) et I engendré par des chemins de Q (on dit que A est une algèbre monomiale). Il est connu que l'extension triviale $T(A)$ est de représentation finie si, et seulement si, la catégorie dérivée D^b (mod A) est équivalente à D^b (mod $k\Delta$) pour Δ un carquois de type Dynkin. Dans ce travail, nous donnons une caractérisation de la docilité de $T(A)$. On démontre que $T(A)$ est docile si, et

seulement si, A est dérivablement docile, c'est-à-dire quand la catégorie répétitive $\hat{A}$ est docile. On donne aussi une caractérisation de la docilité de $T(A)$ par des propriétés du revêtement universel $U_A \to A$.

Théorème. *Soit $A = kQ/I$ une algèbre monomiale triangulaire. Soit $F\colon U_A \to A$ le revêtement universel déterminé par l'action du groupe libre G. Les conditions suivantes sont équivalentes:*

(1) *$T(A)$ est docile ;*
(2) *A est dérivablement-docile ;*
(3) *U_A satisfait à une des conditions suivantes :*
   (i) *$U_A = A$ et A est dérivée équivalente à une algèbre héréditaire de type Dynkin ou Euclidien ou à une algèbre tubulaire ou à une semichaîne $S(n,m)$;*
   (ii) *Le rang $r(G)$ du groupe G est positif et toute sous-catégorie convexe de U_A est dérivée équivalente à une algèbre héréditaire du type $\mathbb{A}_n$;*
   (iii) *$r(G) > 0$ et il existe une suite $(B_n)_n$ de sous-catégories convexes de U_A avec $B_n \subset B_{n+1}$, $\lim_{n\to\infty} B_n = U_A$ et chaque B_n est dérivée équivalente à une semichaîne $S(n,m)$.*

REFERENCES

1. H. Asashiba, *A covering technique for derived equivalence*, J. Algebra **191** (1997), 382–415.
2. I. Assem, D. Happel and O. Roldán, *Representation-finite trivial extension algebras*, J. Pure Appl. Algebra **33** (1984), 235–242.
3. I. Assem and J.A. de la Peña, *On the tameness of trivial extension of algebras*, Fund. Math. **149** (1996), 171–181.
4. I. Assem and A. Skowroński, *Iterated tilted algebras of type $\tilde{\mathbb{A}}$*, Math. Z. **195** (1987), 269–290.
5. I. Assem and A. Skowroński, *On tame repetitive algebras*, Fund. Math. **142** (1993), 59–84.
6. M. Barot, T. Brüstle and J. A. de la Peña, *Derived-tame tree algebras of type $\mathbb{E}$*, Forum Math. **12** (2000), 713–721.
7. T. Brüstle, *Derived tame tree algebras*, Bielefeld (2000) (Preprint).
8. W.W. Crawley-Bœvey, *Functorial filtrations II: Clans and the Gelfand problem*, J. London Math. Soc. (2) **40** (1989), 9–30.
9. P. Dowbor and A. Skowroński, *On Galois coverings of tame algebras*, Arch. Math. (Basel) **44** (1985), 522–529.
10. P. Gabriel, *Auslander-Reiten sequences and representation-finite algebras*, Representation theory, 1 (Proc. Workshop, Carleton Univ. Ottawa, Ont. 1979), 1–71; *Lecture Notes in Math*, vol. 831, Springer, Berlin, 1980.
11. P. Gabriel, *The universal cover of a representation-finite algebra*, Representations of algebras (Puebla, 1980), 68–105; *Lecture Notes in Math.*, vol. 903, Springer, Berlin-New York, 1981.
12. C. Geiss, *On degenerations of tame and wild algebras*, Arch. Math. (Basel) **64** (1995), 11–16.
13. C. Geiss, *Derived-tame algebras and Euler forms* (2000) (Preprint).
14. C. Geiss and J. A. de la Peña, *An interesting family of algebras*, Arch. Math. (Basel) **60** (1993), 25–35.
15. C. Geiss and J. A. de la Peña, *Auslander-Reiten components for clans*, Bol. Soc. Mat. Mexicana (3) **5** (1999), 307–326.

16. D. Happel, *Triangulated categories in the representation theory of finite-dimensional algebras*, London Mathematical Society Lecture Note Series, vol. 119, Cambridge University Press, Cambridge, 1988.
17. D. Hughes and J. Waschbüsch, *Trivial extensions of tilted algebras*, Proc. London Math. Soc. (3) **46** (1983), 347–364.
18. B. Keller, *Algèbres héréditaires par morceaux de type* $\mathbb{D}_n$, C.R. Acad. Sci. Paris Sér. I Math. **312** (1991); no. 7, 483–486.
19. H. Krauze, *Stable module categories and their representation type*, Bielefeld (1996) (Preprint).
20. R. Martínez-Villa and J. A. de la Peña, *The universal cover of a quiver with relations*, J. Pure Appl. Algebra **30** (1983); no. 3, 277–292.
21. J. Nehring, *Polynomial growth trivial extensions of non-simply connected algebras*, Bull. Polish Acad. Sci. Math. **36** (1988), 441–445.
22. C. Novoa and J. A. de la Peña, *On the representation type of certain trivial extensions* (To appear in Proyecciones, Revista de la Universidad Católica de Chile).
23. J.A. de la Peña, *Algebras whose derived category is tame*, Trends in the representation theory of finite-dimensional algebras (Seattle, WA, 1997), 117–127; *Contemp. Math.*, vol. 229, Amer. Math. Soc., Providence, RI, 1998.
24. Z. Pogorzaly and A. Skowroński, *Selfinjective biserial standard algebras*, J. Algebra **138** (1991), 491–504.
25. C.M. Ringel, *Tame algebras and integral quadratic forms*, Lecture Notes in Mathematics, vol. 1099, Springer-Verlag, Berlin, 1984.
26. C. M. Ringel, *The repetitive algebra of a gentle algebra*, Bol. Soc. Mat. Mexicana (3) **3** (1997), 235–253.
27. T. Wakamatsu, *Stable equivalence between universal covers of trivial extension selfinjective algebras*, Tsukuba J. Math. **9** (1985), 299–316.

C. Novoa
Dept. Matemática e Física
Universidad Católica de Goiás
Av. Universitaria 1440/st. Universitario
CEP 74000-000 Goiânia-Go
Brasil
email: cristiannovoa@netscape.net

J.A. de la Peña
Instituto de Matemáticas, UNAM
Circuito Exterior
Ciudad Universitaria
México 04510, D.F.
México
email: jap@penelope.matem.unam.mx