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# A GENERALIZATION OF SYMMETRIC ALGEBRAS

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RÉSUMÉ. Une classe d'algèbres $n$-symétriques est introduite pour chaque entier positif $n$. Il est démontré que, pour toute algèbre $n$-symétrique $A$ et tout $A$-module à gauche sans facteur direct projectif, on a $\tau^n(M) \cong \Omega^{2n}(M)$, où $\tau$ désigne la translation d'Auslander-Reiten et $\Omega$ le foncteur lacet de Heller.

ABSTRACT. For every positive integer $n$ a class of $n$-symmetric algebras is introduced. It is proved that for every $n$-symmetric algebra $A$ and for every left $A$-module $M$ without projective direct summands it holds $\tau^n(M) \cong \Omega^{2n}(M)$, where $\tau$ is the Auslander-Reiten translation and $\Omega$ is the Heller's loop-space functor.

**0. Introduction.** Let $K$ be a fixed algebraically closed field. We use the term algebra to mean finite-dimensional associative $K$-algebra with a unit element. Algebras, as usual in representation theory, are assumed to be basic and connected.

We are interested in self-injective algebras $A$, e.g. $A \cong D(A)$ as left $A$-modules, where $D = \mathrm{Hom}_K(-, K)$ stands for the usual duality. An important class of self-injective algebras is formed by symmetric algebras. This class includes blocks of group algebras. A symmetric algebra can be characterized as an algebra $A$ for which there is an $A$-bimodule isomorphism $A \cong D(A)$.

For any algebra $A$, $(A)\mathrm{mod}$ (resp. $\mathrm{mod}(A)$) mean the category of finite-dimensional left (resp. right) $A$-modules and by $(A)\underline{\mathrm{mod}}$ (resp. $\underline{\mathrm{mod}}(A)$) its stable category modulo projectives. If $A$ is self-injective then there are two interesting equivalences $\Omega : (A)\underline{\mathrm{mod}} \to (A)\underline{\mathrm{mod}}$ and $\tau : (A)\underline{\mathrm{mod}} \to (A)\underline{\mathrm{mod}}$, where $\Omega$ is the Heller's loop-space functor [11] and $\tau$ is the Auslander-Reiten translation [2]. If $A$ is symmetric then for every object $M \in (A)\underline{\mathrm{mod}}$, we have $\tau(M) \cong \Omega^2(M)$.

The aim of this note is to indicate for any positive integer $n$ a class of self-injective algebras for which it holds $\tau^n(M) \cong \Omega^{2n}(M)$. This note is organized in the following way. In Section 1, for any positive integer $n$, an $n$-symmetric algebra is defined. In Section 2, we prove that for any $n$-symmetric algebra $A$ and any $M \in (A)\underline{\mathrm{mod}}$ it holds $\tau^n(M) \cong \Omega^{2n}(M)$. In Section 3, we prove that for any positive integer $n$, there exist $n$-symmetric algebras. Finally, in Section 4, we show that the class of $n$-symmetric algebras is closed under derived equivalences.

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**1. Preliminaries.**

**1.1.** A *quiver* is a pair $Q = (Q_0, Q_1)$, where $Q_0$ is the set of vertices of $Q$ and $Q_1$ is the set of arrows of $Q$ between vertices from $Q_0$. A quiver is said to be *finite* if $Q_0$, and $Q_1$ are finite sets. A *path* in a quiver $Q$ is a sequence of arrows $w = \alpha_1 \dots \alpha_n$, such that for each $i = 1, \dots, n-1$, the target $t(\alpha_i)$ of $\alpha_i$ coincides with the source $s(\alpha_{i+1})$ of $\alpha_{i+1}$. Moreover, to every vertex $x \in Q_0$, we attach a trivial path $e_x$.

For every finite quiver $Q$ we consider its *path algebra* $KQ$ over $K$ [6, 8]. The algebra $KQ$ is a $K$-vector space spanned by the paths in $Q$ (including the trivial ones). The multiplication of basis elements is given by the formula

$$(\alpha_1 \dots \alpha_n)(\beta_1 \dots \beta_m) = \begin{cases} \alpha_1 \dots \alpha_n \beta_1 \dots \beta_m & \text{if } t(\alpha_n) = s(\beta_1) \\ 0 & \text{otherwise}. \end{cases}$$

For every nontrivial path $w = \alpha_1 \dots \alpha_n$ in a quiver $Q$, we denote by $n$ the *length* $l(w)$ of $w$. Moreover, for every trivial path $e_x$ we put $l(e_x) = 0$. Denote by $(KQ)^n$ the two-sided ideal in $KQ$ which is generated by the paths of length at least $n$. A two-sided ideal $I$ in $KQ$ is said to be *admissible* if $I \subset (KQ)^2$ and for each vertex $x \in Q_0$ there is a non-negative integer $n_x$ such that for all $y \in Q_0$ we have $I(x, y) \supset (KQ)^{n_x}(x, y)$, $I(y, x) \supset (KQ)^{n_x}(y, x)$, where for any $z, v \in Q_0$, $KQ(z, v)$ is the $K$-subspace in $KQ$ spanned by the paths sourced at $z$ and targeted at $v$, and $I(z, v) = I \cap KQ(z, v)$.

A *bound quiver* is a pair $(Q, I)$, where $Q$ is a quiver and $I$ is an admissible ideal in the path algebra $KQ$.

**1.2.** It was shown in [6] that for every basic and connected finite-dimensional $K$-algebra $A$ there is a finite connected quiver $Q_A$ and an admissible ideal $I_A$ in $KQ_A$ such that $A \cong KQ_A/I_A$. Thus all considered algebras in this note will be of the form $A = KQ_A/I_A$. An algebra $A \cong KQ_A/I_A$ is said to be *triangular* if $Q_A$ has no oriented cycle.

Let $A = KQ_A/I_A$ be a self-injective $K$-algebra. Consider a map $\pi_A : (Q_A)_0 \to (Q_A)_0$ which sends a vertex $x \in (Q_A)_0$ onto a vertex $y \in (Q_A)_0$ as follows. Let $w$ be a path in $Q_A$ of maximal length with $w \notin I_A$ and $x$ be the source of $w$. Then $y$ is the target of $w$. We infer by [16] that $\pi_A$ is a well-defined map which is bijective.

**1.3. Lemma.** *Let $A = KQ_A/I_A$ be a self-injective $K$-algebra. For every vertex $x \in (Q_A)_0$ there is a positive integer $m_x$ such that $\pi_A^{m_x}(x) = x$.*

*Proof.* For the proof consider an infinite sequence $x = \pi_A^0(x), \pi_A(x), \pi_A^2(x), \dots$ of vertices in $(Q_A)_0$. Since $Q_A$ is a finite quiver, there are minimal non-negative distinct integers $m_1, m_2$ such that $\pi_A^{m_1}(x) = \pi_A^{m_2}(x)$. Since $\pi_A : (Q_A)_0 \to (Q_A)_0$ is bijective, we infer by minimality of $m_1$ that $m_1 = 0$. Hence $m_2$ satisfies $\pi_A^{m_2}(x) = x$. □

**1.4. Proposition.** *Let $A = KQ_A/I_A$ be a self-injective $K$-algebra. Then there is a minimal positive integer $m_A$ such that for every $x \in (Q_A)_0$ it holds $\pi_A^{m_A}(x) = x$.*

*Proof.* We know from Lemma 1.3 that for every vertex $x \in (Q_A)_0$ there is a positive integer $m_x$ such that $\pi_A^{m_x}(x) = x$. Take $m_x$ minimal with this property. Let $m_A$ be the least common multiplicity of all minimal $m_x$, $x \in (Q_A)_0$. Then it is clear that for every $x \in (Q_A)_0$ we have $m_A = c_x \cdot m_x$ and $\pi_A^{m_A}(x) = \pi_A^{c_x \cdot m_x}(x) = (\pi_A^{m_x})^{c_x}(x) = x$. Moreover, minimality of $m_A$ is obvious. □

**1.5.** Let $A = KQ_A/I_A$ be a self-injective $K$-algebra. Then the bijection $\pi_A : (Q_A)_0 \to (Q_A)_0$ induces an isomorphism of left $A$-modules $\mu_A : A \to D(A)$ which is the $K$-linear morphism attaching to every nonzero path $w$ in $Q_A$ the $K$-linear morphism $\mu_A(w) \in \mathrm{Hom}_K(A, K)$ in the following way: if $w$ starts at $x$ then fix a $K$-basis of maximal non-zero paths of the form $vw$ with $v$ ending in $\pi_A(x)$. Then $\mu_A(w)$ is the $K$-linear morphism from $A$ to $K$ such that $[\mu_A(w)](v) = 1$ if $vw$ is an element of the fixed $K$-basis and $[\mu_A(w)](z) = 0$ for every $z \neq v$.

Furthermore, for every $a \in A$ we have a map $\nu_A(a) : A \to A$ given by the formula $\nu_A(a)(b) = \mu_A^{-1}(\mu_A(b) \cdot a)$, $b \in A$. It is easily seen that $\nu_A(a) : A \to A$ is a homomorphism of left $A$-modules. Hence $\nu_A(a) \in \mathrm{Hom}_A({}_AA, {}_AA) = A$ for every $a \in A$. Moreover, for any $b \in A$, we have $\mu_A(b \cdot \nu_A(a)) = \mu_A(b) \cdot a$. Thus $\mu_A(b \cdot \nu_A(aa')) = \mu_A(b) \cdot aa' = \mu_A(b \cdot \nu_A(a)) \cdot a' = \mu_A(b \cdot \nu_A(a) \cdot \nu_A(a'))$. Hence specifying $b = 1$ and applying $\mu_A^{-1}$ one gets that the map $\nu_A : A \to A$ is an automorphism of the algebra $A$. But the map $\mu_A : {}_1A_{\nu_A} \to D(A)$ is an $A$-bimodule isomorphism, where ${}_1A_{\nu_A}$ is an $A$-bimodule with multiplication given by the formula $a * c * b = a \cdot c \cdot \nu_A(b)$, $a, b, c \in A$. We infer by Lemma 12.16 of [5] that for any two automorphisms $\alpha, \beta : A \to A$, there is an $A$-bimodule isomorphism ${}_1A_\alpha \otimes {}_1A_\beta \cong {}_1A_{\alpha\beta}$ given by ${}_1A_\alpha \otimes_A {}_1A_\beta \cong {}_{\alpha^{-1}}A_1 \otimes_A {}_1A_\beta \cong {}_{\alpha^{-1}}A_\beta \cong {}_1A_{\alpha\beta}$, where in the sequence we use the $A$-bimodule isomorphism ${}_\eta A_\delta \cong {}_{\lambda\eta}A_{\lambda\delta}$ given by $x \to \lambda(x)$ for any automorphisms $\eta, \lambda, \delta$ of $A$. Furthermore, ${}_1A_\alpha \cong A$ as $A$-bimodules if, and only if, $\alpha$ is an inner automorphism of $A$, e.g. $\alpha(a) = \lambda^{-1}a\lambda$ for some invertible $\lambda \in A$.

Let $n$ be a positive integer. A self-injective $K$-algebra $A$ is defined to be $n$-*symmetric* if $n = m_A$, where $m_A$ is as in Proposition 1.4, and $\nu_A^n$ is an inner automorphism of $A$. Observe that by Theorem 2.3.1 of [16], we have an equivalence: $A$ is a 1-symmetric algebra if, and only if, $A$ is symmetric.

## 2. The Auslander-Reiten translate for $n$-symmetric algebras.

**2.1.** Let $A = KQ_A/I_A$ be an $n$-symmetric $K$-algebra. Let $\Omega : (A)\underline{\mathrm{mod}} \to (A)\underline{\mathrm{mod}}$ be the Heller's loop-space functor [11]. Recall from [2] that for an algebra $A$ a functor $Tr : (A)\underline{\mathrm{mod}} \to \underline{\mathrm{mod}}(A)$ is defined for objects as follows. For any $M \in (A)\underline{\mathrm{mod}}$ let $P_1 \xrightarrow{f} P_0 \to M \to 0$ be a minimal projective resolution of $M$ in $(A)\mathrm{mod}$. Then $Tr(M) = \mathrm{coker}(\mathrm{Hom}_A(f, A))$. Denote the composed functors $DTr$ by $\tau$ and $TrD$ by $\tau^{-1}$.

**2.2. Lemma.** *If $A$ is a self-injective $K$-algebra, then for any $M \in (A)\underline{\mathrm{mod}}$ we have $\tau(M) \cong D(\mathrm{Hom}_A(\Omega^2(M), A))$.*

*Proof.* It is clear that for any $M \in (A)\underline{\mathrm{mod}}$ there is the following exact sequence in $(A)\mathrm{mod}$ : $0 \to \Omega^2(M) \to P_1 \xrightarrow{f} P_0 \to M \to 0$, where $P_0$, $P_1$ are projective. Applying the functor $\mathrm{Hom}_A(-, A)$ we obtain the following exact sequence in $\mathrm{mod}(A)$ : $0 \to \mathrm{Hom}_A(M, A) \to \mathrm{Hom}_A(P_0, A) \xrightarrow{\mathrm{Hom}_A(f,A)} \mathrm{Hom}_A(P_1, A) \to \mathrm{Hom}_A(\Omega^2(M), A) \to 0$. Thus $\mathrm{coker}(\mathrm{Hom}_A(f, A)) \cong \mathrm{Hom}_A(\Omega^2(M), A)$. Hence $Tr(M) \cong \mathrm{Hom}_A(\Omega^2(M), A)$, and so $\tau(M) = DTr(M) \cong D(\mathrm{Hom}_A(\Omega^2(M), A))$. □

**2.3. Lemma.** *If $A$ is a self-injective $K$-algebra and $M \in (A)\underline{\mathrm{mod}}$ then $\tau\Omega^{-2}(M) \cong M \otimes_A {}_1A_{\nu_A}$.*

*Proof.* We know from [6] that the functor $D(\mathrm{Hom}_A(-,A))$ is naturally equivalent to $-\otimes_A D(A)$. Thus $\tau\Omega^{-2}(M) \cong D(\mathrm{Hom}_A(M,A))$ by Lemma 2.2. Then $\tau\Omega^{-2}(M) \cong M \otimes_A D(A)$. But we know from 1.5 that $D(A) \cong {}_1A_{\nu_A}$ as $A$-bimodules, hence $\tau\Omega^{-2}(M) \cong M \otimes_A {}_1A_{\nu_A}$. □

**2.4. Theorem.** *Let $A = KQ_A/I_A$ be an $n$-symmetric $K$-algebra. Then for any $M \in (A)\underline{\mathrm{mod}}$ it holds $\tau^n(M) \cong \Omega^{2n}(M)$.*

*Proof.* We infer by Lemma 2.3 that

$$(\tau\Omega^{-2})^n(M) \cong M \otimes_A \underbrace{{}_1A_{\nu_A} \otimes \cdots \otimes {}_1A_{\nu_A}}_{n\times}.$$

But we know from 1.5 that

$$\underbrace{{}_1A_{\nu_A} \otimes \cdots \otimes {}_1A_{\nu_A}}_{n\times} \cong {}_1A_{\nu_A^n} \cong A$$

as $A$-bimodules, because $A$ is $n$-symmetric. Thus $(\tau\Omega^{-2})^n(M) \cong M$. Since the functors $\tau$, $\Omega$ commute for the objects [3, 4], $\tau^n(M) \cong \Omega^{2n}(M)$. □

## 3. Examples of $n$-symmetric algebras.

**3.1.** Let $B$ be a finite-dimensional $K$-algebra which is triangular. Following [12] and [15] the *repetitive algebra* $\hat{B}$ of the algebra $B$ is a self-injective, locally finite-dimensional matrix algebra without unit

$$\hat{B} = \begin{bmatrix} \ddots & 0 & 0 & \cdots & 0 \\ \ddots & B_{-1} & 0 & \cdots & 0 \\ & E_{-1} & B_0 & 0 & \cdots \\ \cdots & 0 & E_0 & B_1 & 0 \\ 0 & \cdots & 0 & \ddots & \ddots \end{bmatrix},$$

where matrices have only finitely many non-zero coefficients and for every integer $i$, $B_i = B$, $E_i$ is the minimal injective cogenerator $E = D(B)$. The addition is the usual matrix addition, and the multiplication is induced by the canonical $B$-bimodule structure on $E$ and the zero map $E \otimes_B E \to 0$ [12]. Let $\nu_{\hat{B}}$ be the Nakayama automorphism of the algebra $\hat{B}$ which is induced by $\pi_{\hat{B}}$ from 1.5. Then for every positive integer $n$ the infinite cyclic group $(\nu_{\hat{B}}^n)$ generated by $\nu_{\hat{B}}^n$ acts freely on $\hat{B}$ in such a way that the quotient category $\hat{B}/(\nu_{\hat{B}}^n)$ in the sense of [6] is a finite-dimensional self-injective $K$-algebra with a unit element.

Let $\hat{B} = KQ_{\hat{B}}/I_{\hat{B}}$ for some infinite quiver $Q_{\hat{B}}$ and an admissible ideal in the path category $KQ_{\hat{B}}$. Then for any vertex $x \in (Q_{\hat{B}})_0$ we have $\nu_{\hat{B}}(e_x) = e_{\pi_{\hat{B}}(x)}$ and for any arrow $\alpha$ in $Q_{\hat{B}}$, $\nu_{\hat{B}}(\alpha)$ is an arrow in $Q_{\hat{B}}$. Thus for any path $w$ in $Q_{\hat{B}}$, the image $\nu_{\hat{B}}(w)$ is a path in $Q_{\hat{B}}$.

**3.2. Theorem.** *The algebra $\hat{B}/(\nu_{\hat{B}}^n)$ is $n$-symmetric and is not $(n-1)$-symmetric.*

*Proof.* It is well-known that there is a Galois covering functor $F : \hat{B} \to \hat{B}/(\nu_{\hat{B}}^n)$ induced by the free action of the group $(\nu_{\hat{B}}^n)$ on $\hat{B}$. Moreover, the group $(\nu_{\hat{B}}^n)$ acts freely on the quiver $Q_{\hat{B}}$ in such a way that there is a quiver $Q$ and an admissible ideal $I$ in $KQ$ such that $\hat{B}/(\nu_{\hat{B}}^n) \cong KQ/I = A$ and every $(\nu_{\hat{B}}^n)$-orbit of a vertex $x$ in $Q_{\hat{B}}$ is mapped onto a vertex $F(x)$ in $Q$. Moreover, every $(\nu_{\hat{B}}^n)$-orbit of an arrow $\alpha$ in $Q_{\hat{B}}$ is mapped onto an arrow $F(\alpha)$ in $Q$. It is also well-known that every $(\nu_{\hat{B}}^n)$-orbit of a vertex in $Q_{\hat{B}}$ consists only of vertices and every $(\nu_{\hat{B}}^n)$-orbit of an arrow in $Q_{\hat{B}}$ consists only of arrows. Thus for every vertex $x \in (Q_{\hat{B}})_0$ it holds $\pi_A(F(x)) = F(\pi_{\hat{B}}(x))$. If we choose the induced morphisms $\mu_A$ and $\nu_A$ in such a way that for every arrow $\alpha \in (Q_{\hat{B}})_1$ it holds $\nu_A(F(\alpha)) = F(\nu_{\hat{B}}(\alpha))$ then $\nu_A^n = \mathrm{id}_A$ and $\pi_A^{n-1} \neq \mathrm{id}_A$. Consequently, the algebra $A$ is $n$-symmetric and is not $(n-1)$-symmetric. $\square$

**4. Derived equivalences of $n$-symmetric algebras.**

**4.1.** We shall freely make use of results on triangulated and derived categories from [9, 10, 13]. We shall use the notations of [13].

Given a $K$-algebra $A$, denote by $\mathrm{Mod}(A)$ the category of all right $A$-modules. Consider the derived category $D^-(\mathrm{Mod}(A))$ in the sense of Hartshorne [10]. For any object $X^* \in D^-(\mathrm{Mod}(A))$ denote by $\tilde{\nu}_A X^*$ the object $X^* \otimes_A^L D(A)$ of $D^-(\mathrm{Mod}(A))$. Rickard proved in [13] that for any derived equivalence $F : D^-(\mathrm{Mod}(A)) \to D^-(\mathrm{Mod}(B))$ we have $F(\tilde{\nu}_A X^*) \cong \tilde{\nu}_B(FX^*)$ for every object $X^*$ of $D^-(\mathrm{Mod}(A))$.

**4.2.** It was proved in [13] that if $F : D^-(\mathrm{Mod}(A)) \to D^-(\mathrm{Mod}(B))$ is a derived equivalence then there is an object $\Delta^*$ and $\Theta^*$ of $D^b(\mathrm{Mod}(B^{op} \otimes A))$ and $D^b(\mathrm{Mod}(A^{op} \otimes B))$ such that $F = - \otimes_A^L \Delta^*$ and $\Delta^* \otimes_B^L \Theta^* \cong {}_A A_A$ and $\Theta^* \otimes_A^L \Delta^* \cong {}_B B_B$.

**4.3. Theorem.** *Let $A$ and $B$ be two basic and connected $K$-algebras which are derived equivalent. Then $A$ is n-symmetric if, and only if, $B$ is n-symmetric*.

*Proof.* Suppose that $A$ and $B$ are two basic and connected $K$-algebras which are derived equivalent, e.g. $D^b(\mathrm{Mod}(A))$ and $D^b(\mathrm{Mod}(B))$ are equivalent as triangulated categories. We infer by Theorem 1.1 of [13] that $D^-(\mathrm{Mod}(A))$ and $D^-(\mathrm{Mod}(B))$ are equivalent as triangulated categories.

Suppose that $A$ is n-symmetric. Then for every object $X^*$ of $D^-(\mathrm{Mod}(A))$ we have $(\tilde{\nu}_A)^n X^* \cong X^*$ by 4.1, Lemma 2.3 and 1.5. Moreover, we know from 4.2 that there is a triangulated equivalence $F : D^-(\mathrm{Mod}(A)) \to D^-(\mathrm{Mod}(B))$ given by $F = - \otimes_A^L \Delta^*$. Furthermore, we infer by 4.1 that for every object $Y^*$ of $D^-(\mathrm{Mod}(B))$ we have $(\tilde{\nu}_B)^n Y^* \cong Y^*$ and $n$ is the least number with this property. Thus we infer by Lemma 2.3 that $\nu_B^n$ is an inner automorphism of $B$, because

$$\underbrace{{}_1 B_{\nu_B} \otimes \cdots \otimes {}_1 B_{\nu_B}}_{n\times} \cong B$$

as $B$-bimodules by 4.1 and 4.2. Consequently, $B$ is an $n$-symmetric algebra.

Replacing $A$ and $B$ we obtain the converse implication and our theorem is proved. $\square$

**4.4. Remark**. Applying Theorem 4.3 one can easily obtain the main results of Sections 4 and 5 of [1] without the need to apply covering techniques to derived categories.

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**Résumé substantiel en français.** Soit $K$ un corps algébriquement clos. Toutes les algèbres considérées ici sont associatives, unifères et de $K$-dimension finie. Toute algèbre auto-injective $A$ donne lieu à deux équivalences intéressantes $\Omega : (A)\underline{\text{mod}} \to (A)\underline{\text{mod}}$ et $\tau : (A)\underline{\text{mod}} \to (A)\underline{\text{mod}}$, où $\Omega$ désigne le foncteur lacet de Heller et $\tau$ la translation d'Auslander-Reiten.

Si $A$ est symétrique, alors, pour chaque objet $M \in (A)\underline{\text{mod}}$, on a $\tau(M) \cong \Omega^2(M)$. Soit $A = KQ_A/I_A$ une $K$-algèbre auto-injective donnée par un carquois lié $(Q_A, I_A)$. Il existe alors un plus petit entier positif $m_A$ tel que, pour chaque sommet $x$ de $Q_A$, on ait $\pi_A^{m_A}(x) = x$, où $\pi_A$ désigne la permutation de Nakayama qui induit l'automorphisme de Nakayama $\nu_A : A \to A$.

Pour chaque entier positif $n$, une $K$-algèbre auto-injective $A$ est dite $n$-symétrique si $n = m_A$ et $\nu_A^n$ est un automorphisme interne de $A$.

Évidemment, une algèbre est symétrique si et seulement si elle est 1-symétrique.

Les théorèmes suivants sont les résultats principaux du présent travail.

**Théorème 1.** *Soit $A = KQ_A/I_A$ une $K$-algèbre $n$-symétrique. Alors, pour chaque $M \in (A)\underline{\text{mod}}$, on a $\tau^n(M) \cong \Omega^{2n}(M)$.*

**Théorème 2.** *Soit $\hat{B}$ l'algèbre répétitive d'une algèbre triangulaire $B$, alors l'algèbre $\hat{B}/(\nu_{\hat{B}}^n)$ est $n$-symétrique, mais non $(n-1)$-symétrique.*

**Théorème 3.** *Soit $A$ et $B$ deux $K$-algèbres sobres et connexes qui sont dérivées-équivalentes. Alors $A$ est $n$-symétrique si et seulement si $B$ est $n$-symétrique.*

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